Analysis of a MIMO Outdoor Channel with Hybrid EM-based Modeling

Shirook Ali, Farzaneh Kohandani, and Geyi Wen

Advanced Technology, Research In Motion Ltd.
185 Columbia St. W., Waterloo, ON., N2L 5Z5, Canada

Abstract — **We consider the effects of the local scatterers, the number of antennas, the mobile position, and the signal-to-noise ratio on the performance of a multi-antenna system via electromagnetic–based modeling. The urban microcell scenario is adopted and simulated using a hybrid electromagnetic model to obtain the channel matrix. Comparisons with the measurement-based spatial channel model (SCM) at frequency 2 GHz are made. We show that the channel condition improves as the multipath richness enhances. This also improves the capacity, which becomes more pronounced as the number of antennas increases. Our discussions show that the SCM does not reflect the actual system performance in some scenarios.**

Index Terms — **MIMO, Electromagnetic (EM) Modeling, Urban Microcell, Spatial Channel Model (SCM).**

I. INTRODUCTION

There has been considerable attention to multi antenna systems known as Multi-Input Multi-Output (MIMO) due to their unique features [1,2]. These features promise to outperform the traditional Single-Input Single-Output (SISO) systems in transmission rate and/or signal quality.

Early investigations on the capability of MIMO systems are from contributions in the information theory community [1,2]. The majority of these studies make many simplifications in their assumptions, resulting in the lack of preciseness in their scenarios. Hence, the evaluation of the MIMO system is not realistic and its actual capability is not known. A realistic evaluation requires a complete picture of the system, which includes the antenna and the detailed channel. Thus, the electromagnetic (EM) viewpoint is essential in the MIMO system evaluation.

The importance of studying MIMO from an EM perspective has recently become noticeable [3]-[7]. For example, the effect of the mutual coupling between the antennas is currently known to have a significant effect on the MIMO system performance. Mutual coupling has been investigated through measurements [6], theoretical results [7], and simulations [5].

However, it has been noted [3] that the scattering models used to arrive at these conclusions are often simplified and not much detail is given. The mobile position is usually assumed to be either in a line-of-sight position or in a non line-of-sight position; the performance as the position changes is not studied.

In this work, we analyze the performance of the MIMO system in an urban microcell outdoor channel. The effects of the local scatterers[1], the number of antennas, the position of the mobile station (MS), and the signal-to-noise ratio (SNR) on the performance of the system are studied. Our analyses are from a complete EM point of view. The analyses are made on the basis of the principles of electromagnetic wave propagation and antenna theory.

The EM-based model is compared to the measurement-based spatial channel model (SCM) [9]. We extend the analyses to include scenarios where the SCM might not reflect the actual system performance.

II. EM-BASED AND SCM CHANNEL MODELS

Since the employed EM-based channel model is deterministic[2], the location of the scatterers/reflectors or the direction of the multipath components at the transmitter and at the receiver are given and are exactly simulated. This model is compared to the probabilistic[3] measurement-based SCM channel model.

A. EM-based Channel Model (hybrid MoM/UTD)

A general linear time invariant MIMO system with M inputs and N outputs is assumed. The modeled system is shown in Fig. 1. The channel includes everything within the borders of the dashed line. The left and right borders of this boundary correspond to the antennas' reference plane on the transmit and the receive sides, respectively.

The input signal vector $\boldsymbol{x}$ and the output signal vector $\boldsymbol{y}$ are related through the channel matrix $\boldsymbol{H}$ after adding the white Gaussian noise vector $\boldsymbol{z}$,

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[1] The effect of the far scatterers is studied in [8].

[2] Deterministic model results the solution for a given set of data

[3] Probabilistic model can result the average solution to all the realizations in the ensemble.

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{z} \, . \tag{1}$$

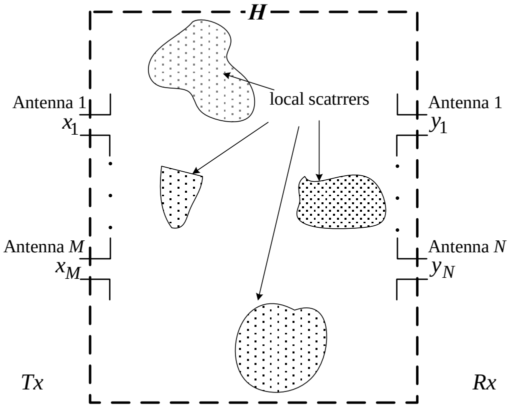

Fig. 1. An arbitrary N×M MIMO system including the local scatterers and showing the boundaries of the channel matrix $\boldsymbol{H}$.

The transmitted signal from the individual antennas are independent and equi-powers $\sqrt{p_t/M}$, where p_t is the total transmitted power. p_t is constraint by the covariance matrix[4] $\boldsymbol{R_{xx}} = E\left\{\boldsymbol{xx}^H\right\}$, satisfies $\mathrm{Tr}\left(\boldsymbol{R_{xx}}\right) = M$.

The elements of the channel matrix $\boldsymbol{H}$ are related to the transmission coefficients between the transmit and the receive antennas of the system [4]. It is computed as

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & \cdots & h_{1M} \\ \vdots & \ddots & \vdots \\ h_{N1} & \cdots & h_{NM} \end{bmatrix}, \; h_{ij} = S_{ij}\Big|_{i \neq j} \tag{2}$$

where S_{ij} , $i = 1, \ldots, N; \; j = 1, \ldots, M$, is the complex transmission coefficient between the ith transmit antenna and the jth receive antenna. Each element of $\boldsymbol{H}$ can be decomposed into two main components

$$S_{ij} = S_{ij}^d + S_{ij}^m \, . \tag{3}$$

In (3), S_{ij}^d is the direct component; S_{ij}^m is the superposition of all the multipath components, ξ_m . S_{ij}^m is a function of the medium parameters, the EM field, the frequency of operation, the normalized incident vectors at the antenna excitation, the number of scatterers N_s , and the volume per scatterer V_m – see [4] for details,

$$S_{ij}^m = \frac{1}{\alpha} \sum_{n=1}^{N_s} \int_{V_m} \xi_{m,n} \, . \tag{4}$$

The elements of the channel matrix are computed via the EM-based simulation. We use a hybrid MoM/UTD simulation. This provides flexibility to model electrically large problems yet preserve the accuracy where needed. The antennas and their responses are modeled via the MoM. The rest of the problem is modeled via the UTD.

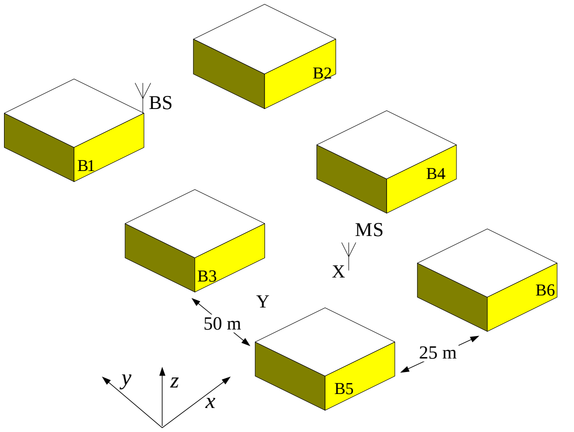

Fig. 2:The simulated microcell scenario in the EM model – BS is at height 12.5 m and MS is at hiegth1.5 m.

B. Probabilistic-based Channel Model (SCM)

The SCM is a measurement-based model. It best fits our simulation scenario compared to other MIMO channel models [9]. It results the average of the many realizations. In simulations, we noticed that the individual realization may significantly vary from the model average. It, therefore, might not reflect the actual estimate for a precise scenario, e.g., as the MS position changes.

III. THE URBAN MICROCELL ENVIRONMENT

The EM simulations are performed using FEKO [11] for the deterministic urban microcell scenario shown in Fig. 2. This scenario is a duplicate of the SCM model [9] where six main paths with twenty sub-paths each are assumed. Referring to (4), six scatterers are considered, i.e., N_s =6, and the twenty sub-paths $\xi_{m,n}$ are considered within the volume integral V_m . These are modeled in the EM model as six 12 m-height buildings (B1 – B6), see Fig. 2 for details. The BS and MS antennas are vertically polarized half-wavelength dipoles operating at 2 GHz. The separation between the dipoles on one side for the MIMO system is half a wavelength.

For each simulation, the system matrix $\boldsymbol{H}$ is computed using (2)-(4). In the general case where channel state information is absent at the transmitter, it is reasonable to choose $\boldsymbol{x}$ to be specially white, i.e., $\boldsymbol{R_{xx}} = \boldsymbol{I}_M$. The mutual information (capacity) achieved is then [1]:

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[4] E is the expected value of the quantity inside the brackets, and Tr is the trace of the quantity between the brackets.

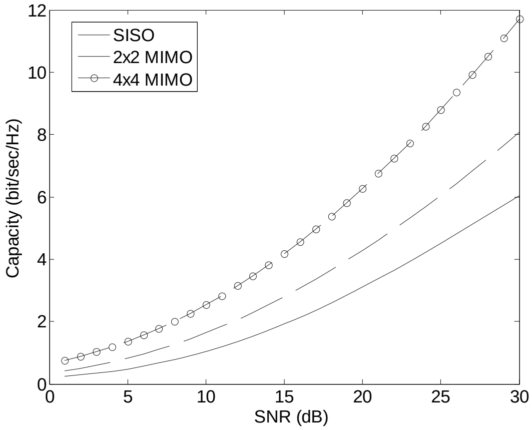

Fig. 3. Mean capacity results for the Urban microcell when the MS is at position X based on the SCM.

$$C = \log_2 \left\{ \det \left[\boldsymbol{I} + \frac{SNR}{M} \boldsymbol{HH}^{\dagger} \right] \right\}. \qquad (5)$$

Here, $\boldsymbol{I}$ is an $M \times M$ identity matrix and $\dagger$ is the conjugate transpose of the corresponding matrix. The SNR is defined here at the transmitter side.

Notice that in the EM-based model, the received single power is exactly know, and the SNR is calculated at the receiver. On the other hand, in (5) and in the SCM, the SNR is computed at the transmitter side. Hence, a comparison between the two models would not be consistent. To compensate for this inconsistency, we introduce a factor of p_t / p_r in (3) in our EM model.

IIII. SIMULATION RESULTS

The MIMO system is evaluated through the channel matrix, the weight and number of independent sub-channels, as well as the information capacity of the system.

The channel matrix is decomposed using the SVD replacing (1) by

$$\boldsymbol{y} = \boldsymbol{U\Sigma V}^{\dagger} \boldsymbol{x} + \boldsymbol{z} . \qquad (6)$$

In (6), $\boldsymbol{U}$ and $\boldsymbol{V}$ are $N \times N$ and $M \times M$ unitary eigenvector matrices, respectively. $\boldsymbol{\Sigma} = \mathrm{diag}\left(\sqrt{\sigma_1}, ..., \sqrt{\sigma_{ij}}, ..., \sqrt{\sigma_{\min(N,M)}}, 0, ..., 0\right)$, where $\sigma_1,..., \sigma_{\min(N,M)}$ are the nonzero eigenvalues. The SVD shows that the channel matrix $\boldsymbol{H}$ can be diagonalized to a number of independent orthogonal sub-channels, where the power gains of the *ij*th sub-channel is σ_{ij} .

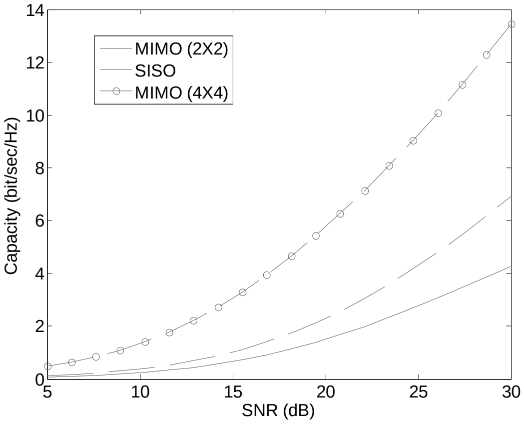

Fig. 4. Capacity results for the Urban microcell when the MS is at position X based on EM modeling.

The results of the simulated urban microcell are given in Figs. 3-6 and Table I. Figs. 3 and 4 show the results based on the SCM for the average of 100K realizations, and the EM-based model, both for the case when the MS is at position X, see Fig. 2. The correspondence between the two results is close despite the different approach applied to obtain each.

We next study the variation in the capacity as the MS moves to position Y between B3 and B5, see Fig. 2. The results based on our-EM model are given in Fig 5. Results based on the SCM show negligible differences with the results when the MS is at position X given in Fig. 3 – they are eliminated here due to space limitations. This negligible difference is expected since the SCM model is a probabilistic based model that produces the average of a large number of realizations. On the other hand, the EM model gives a more realistic view by reflecting the actual effects when the MS's position changes.

Notice that more capacity is obtained when the MS is at position X. This can be explained by looking at (2)-(3). At position X, the direct component S_{ij}^{d} is significant, which improves the coupling/transmission between the transmitter and the receiver. Position X also provides more possibility for additional multipath than position Y. This in turn improves the transmission coefficient S_{ij} .

The effect of the number of scatterers N_s and the number of antennas on the capacity at SNR 30 dB is shown in Fig. 6. Increasing the number of scatterers in this scenario improves the capacity for the MIMO system. From (3)-(4), as the number of scatterers increases, the possibility of having additional multipath components

$\xi_{m,n}$ increases. The improvement is more pronounced as the number of antennas increase.

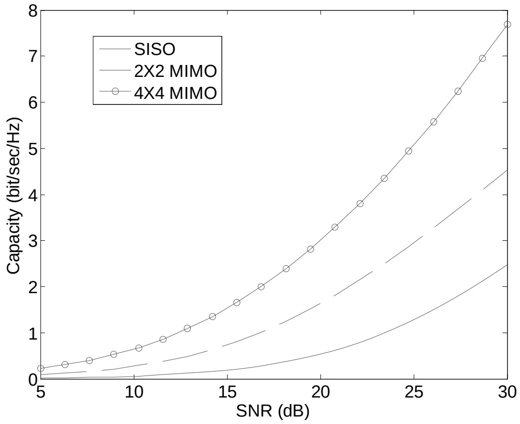

Fig. 5. Capacity results when the MS is at position Y based on the EM model.

TABLE I: THE CHANNEL MATRIX SINGULAR VALUES AND THE CONDITION NUMBER FOR DIFFERENT MIMO CONFIGURATIONS.

Configuration		Σ (dB)	κ
2 × 2	Ns=6, Y	[-29 -41.8]	18
	Ns=6, X	[-28.7 -33.1]	2.7
	Ns=4, X	[-28.7 -33.3]	28.9
	Ns=8, X	[-28.9 -33.6]	33.4
4 × 4	Ns=6, Y	[-27 -33.5 - 45.7 -49.8]	180
	Ns=6, X	[-26 -27.6 -34 -42.7]	42
	Ns=4, X	[-26 -28.6 -36.6 -42.7]	45
	Ns=8, X	[-26 -28 -33.8 -40.3]	25

Table I shows the singular values and the condition number κ of the channel matrix ***H***, see (6), for the different scenarios analyzed above. The values show that the channel matrix when the MS is at position X is better conditioned than when the position is changed to Y. Increasing the number of scatterers provides more multipath richness improving the channel matrix condition. This implies that the power allocation between the individual sub-channels is more proportional. Notice that a change in the setting of the channel has a stronger effect on the performance as the number of antennas increases.

V. CONCLUSION

We present a thorough analysis of an urban microcell outdoor channel for the multi-antenna system using hybrid electromagnetic-based modeling. Our analyses are based on the principles of electromagnetics. The effects of the environment on the channel condition and the information capacity are studied. Increasing the number of scatterers improves the channel matrix condition and the information capacity. This improvement is proportionally pronounced with the number of antennas. Changing the position of the MS can result in a significant change in the system performance. Comparisons with the probabilistic-based SCM are made. These show that the SCM may not reflect the actual system performance in some scenarios.

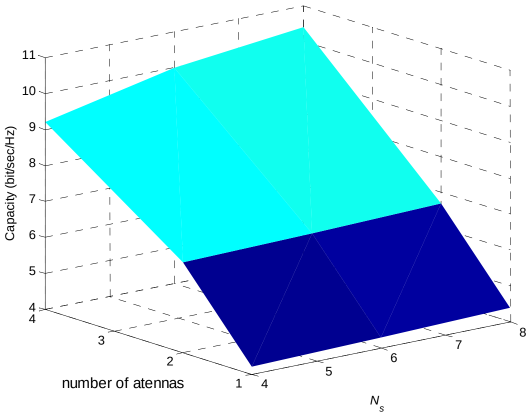

Fig. 6. Effect of the number of antennas and scatterers on the capacity at SNR 30 dB.

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